Nuclear physics

Proton- electron hypothesis :

Until the discovery of neutron this hypothesis was proposed. The fact that certain radioactive elements emit alpha and beta particles both of them are corpuscular in nature led to the idea that atoms are build up of some elementary constituents. The main experimental facts that led to this hypothesis are;

1. Whole number rule:

The masses of different nuclei were whole number multiple of mass of different hydrogen. The hydrogen nuclei was given the name proton ($_1H^1$). The slight change from whole number was explained by the presence of two or more isotopes in which the atomic weight of each isotope is an integer but they are present in different percentage. This leads to presence of protons inside the nucleus.

2. Beta ray decay :

As electrons were known to be emitted from the nuclei so it was believed that electrons were constituents of nucleus and the beta ray emission confirms the existence of electrons inside nucleus.

3. If A is the atomic weight of nucleus it was assumed that nucleus would consist of A protons then charge on nucleus should be A not Z. So it was assumed that in addition to A protons in nucleus there are A-Z electrons. This will contribute negligible mass to the nucleus and makes +Z charge as required.

Contradictions to proton electron hypothesis

- Finite size : each electron is spherical in shape and have finite dimensions so in heavy nuclei it is difficult to have large number of nucleons in a small nucleus.
- Spin consideration: Both proton as well as electron possess half (1/2) spin so the nucleus has *integral spin if p+e= even*

  and *half integral if p+e=odd . But exptlly found that spin depends upon mass number A.*

Spin=0 or integral for even A

And spin=1/2 for odd A.

Example $_7N^{14}$ the number of nucleons inside nucleus =2A-Z =21

Hence such a nucleus should have half integral spin which is against observed fact I=1.

- Magnetic moment nuclear magnetic moment not explained.

Proton neutron hypothesis

- After the discovery of neutron by Chadwick
- Heisenberg assumed that there are protons and neutrons present inside the nucleus.
- The total number of protons and neutrons inside the nucleus is equal to mass number A.
- Number of protons is equal to atomic number Z of atom.
- N represents number of neutrons inside nucleus.
  - Then, $A=Z+N$ or $N=A-Z$
  - As atom as a whole is neutral therefore there are as many as electrons outside nucleus as many protons are present inside nucleus.
  - The total charge of nucleus is equal to charge of all protons present in it.
  - Mass of nucleus is equal to sum of masses of all protons and neutrons in it.

- This theory was widely accepted as N and P are similar in many respects and removed many descrepancies.
- For low nuclei N<Z
- For heavy nuclei N>Z e.g $_{92}U^{238}$
- For N=Z nuclei are almost stable.

Contradictions removed

- Finite size: Since mass of neutron is same as mass of proton and total number of nucleons as same as atomic number. Hence it is not difficult to have neutrons in it.
- Spin: nucleons possess spin and orbital angular momentum i.e., J=l+s.
- Both neutron and proton have spin quantum number ½.

hence spin I is an integer for P+N= A= even

And I=half integral for odd A. Decent agreement with expt results.

- Statistics : neutrons and protons are fermions and thus obey F-D statistics.
  - If number of fermions inside nucleus is even then spin is integral thus obeying B-E statistics and if A is odd then it will obey F-D statistics and is found experimentally true.

- De Broglie wavelength : As $m_p=m_n$ then de-Broglie wavelength for proton or neutron is $\lambda=h/p$

  $\Lambda=0.14$ nuclear diameter

Here wavelength of proton or neutron is comparable to nuclear diameter. Hence protons or neutrons can reside inside the nucleus.

- Β emission : although there is no electron inside the nucleus but in β decay a neutron changes into a proton emitting an e^- and anti-neutrino or a proton changes into a neutron emitting positrion e^+. Thus it could explain dual β decay.
- As $m_n \sim m_p$ also magnetic moment of proton and neutron isn't very much different and both have magnetic moments of order of magnetic moment of nucleus.

General properties of Nucleus

- **Nuclear mass:**
- Mass of nucleus is equal to mass of nucleus plus mass of orbital electrons.
- But $m_e < m_p$. So mass of nucleus is almost equal to atomic mass.
- Nucleus accounts for 99.75% of mass of atom.
- Nuclear mass is measured with good accuracy by mass spectrograph by measuring e/m ratio, it comes out to be very close integral value A.
- The mass of nucleus is measured in atomic mass unit. One a. m. u is equal to one twelfth of the mass of the mass of neutral Carbon-12 atom.
- Since N (Avagadros no.) atoms of Carbon has mass =12g.
- So 1 atom of carbon has mass =12/N.
- 1 a. m.u $= \frac{1}{12}\frac{12}{N}$ g $= \frac{1}{6.023\times10} g = 1.66\times 10^{-24}$ g
- 1 a.m.u$=1.66\times 10^{-27} kg$

Unit of energy

- The unit in which nuclear energy is expressed is called as electron volt (eV).
- The amount of energy acquired by an e^- when accelerated through a potential difference of 1 volt is called an electron volt.
- Thus 1 eV=$1.6\times 10^{-19} C \times 1V = 1.6\times 10^{-1}$J.
- Bigger units are 1 keV=10^3eV.
- 1 MeV=10^6eV.

Relation between energy and a.m.u

- According to Einstein's mass energy equivalence

  $E=mc^2$

Where c is velocity of light=$2.99\times 10^{8} m/s$

Here, m=1 a.m.u $=1.6\times10^{-27}$ kg

Then energy eqvivalent to 1 a.m.u will be $E=1.66\times 10^{-27} \times 9 \times 10^{16} J$

$$E= 1.66\times 10^{-10} J$$

$$E= \frac{1.484\times10^{-10}}{1.6\times10^{-19}} eV$$

$$E=931.484\times 10^{6}\, eV$$

1 a.m.u=931.48 MeV

Radius of nucleus/ Nuclear size

- Nucleus is positively charged and positive charge inside nucleus is spherical or nearly spherical.
- Volume of nucleus is proportional to the number of nucleons i.e., mass number A.
- Volume of nucleus is directly proportional to mass number.

$$V \alpha A$$

$$\frac{4}{3}\Pi r^3 \alpha A$$

$$r \alpha A^{1/3}$$

$$\text{Or } r=r_0 A^{1/3}$$

Where r_0 is constant for all nuclei. Its value varies from 1.1-1.6fm

Where $1\text{fm}=10^{-15}\text{m}=10^{-13}\text{cm}$.

Femtometer- used to describe the dimensions of an atomic nucleus.

Nuclear Density

- The quantity which describes both mass and size of nucleus is called density. Nuclear density is defined as mass per unit volume of nucleus . Thus,

$$\text{Density} = \frac{mass\ of\ nucleus}{volume\ of\ nucleus}$$

Since $r \sim r_0 A^{1/3}$

Then volume $V = 4/3 \Pi r^3$

$V = 4/3 \Pi r_0^3 A$

Mass of nucleus $= A \times 10^{-27} \times 1.67 kg$

Nuclear density $\rho = \frac{A \times 1.67 \times 10^{-27}}{4/3 \pi r_0^3 A} \sim 10^{17} kg/m^3$.

It is clear nuclear density is independent of mass number A. Therefore density of all nuclei is same.

Nuclear charge

The nucleus consists of protons and neutrons. As protons are positively charged and neutrons are neutral, the total charge of nucleus is the integral multiple of proton charge i.e., Ze where z is the number of protons.

Mass defect

- The expected mass of nucleus is equal to the sum of masses of its constituents.
- If there are Z number of protons and N number of neutrons inside nucleus then expected mass = $Zm_p + Nm_N$

  Where m_p is masss of proton and m_n is mass of neutron.

But the actual mass m(Z,A) of nucleus is always less than expected mass i.e., $M(Z,A) < Zm_p + Nm_N$

The difference in these two masses is called as mass defect.

Thus, difference in total mass of nucleus and real mass of that nucleus is called the mass defect

$$\Delta M = Zmp + NmN - M(Z,A)$$

Since N=A-Z

$$\Delta M = Zmp + (A-Z)mN - M(Z,A)$$

Greater the mass defect in the formation of nucleus, more stable is the nucleus.

Packing fraction

- The mass number A(atomic weight) is an integer but the actual mass is not an integer . The deviation of mass of nucleus from mass number A is expressed in terms of a quantity called as packing fraction.
- P.F= $\frac{actual\ mass - mass\ number}{mass\ number}$
- P.F= $\frac{M\text{-}A}{A}$
- M=A[1+p]
- P measures the stability of nucleus
- (M-A) is called mass excess, thus packing fraction is the quantity that tells us the mass excess per nucleon of the nucleus.

Binding energy

- The actual mass of nucleus is less than the mass number A.
- That is actual mass is less than the sum of masses of constituent particles.
- If Δm is difference of mass between mass of nucleus and masses of nucleons. Then using mass-energy equivalence, the energy corresponding to Δm mass is $E=mc^2$
- This much amount of energy is released in the process of formation of nucleus. This energy is called as binding energy.
- Thus mass defect is converted into energy while nucleons combine to form nucleus. The same amount of energy is required to break nucleus into its constituents . Thus mass defect is a measure of binding energy .

- Binding energy of a nucleus is the energy released in the formation of nucleus by combining its nucleons.
- In other words the energy required to disintegrate a nucleus into its constituent particles.
- Binding energy is the measure of stability of nucleus.
- Since $\Delta M = Zmp + \ (A - Z)$ m_N-M(A,Z) . Therefore binding energy , by using mass energy relation,

  $B=[Zm_p+(A-Z)m_N-m(A,Z)]C^2$

In terms of a.m.u the B.E can be written as;

$B=[Zm_p+(A-Z)m_N- M(A,Z)]$a.m.u.

Since we know that 1 a.m.u=931.14MeV

$B(MeV)=931.14\ [Zm_p+(A-Z)m_N- M(A,Z)]$

Binding energy per nucleon

- The average binding energy per nucleon is obtained by dividing total binding energy of nucleus by mass number A.
- Thus amount of energy required to release a nucleon from nucleus is called as B.E per nucleon.
- B’’=$\frac{B}{A}$ =Z/A mp+ m_N (A-Z)/A -M/A
- For example binding energy of deutron=2.225 MeV. Deutron contains two nucleons, the B.E per nucleon is 2.2225/2=1.1125 MeV.
- B.E of $_2He^4$=28 Mev
- B.E/ nucleon is 28/4= 7MeV.

Stability of nucleus on the basis of binding energy and binding energy curve

- Less the B.E/nucleon less is stability of nucleus where as if B.E/nucleon is higher more is stability. Thus Helium is more stable than deutron.
- Upto A=20 B.E/A increases, the curve has got variation upto A=16. The nuclei He, C, O indicate B.E/A is higher than adjoining nuclei. They show extra stability. This is because these nuclei are considered to possess magic number i.e., mass number divisible by 4. These nuclei possess equal pairs of neutrons and protons.
- The B.E first increases sharply and attains maximum value for A=56 of 8.8 MeV.
- B.E/A is positive for all vaues of mass number it means that all nuclei are stable i.e., nuclear forces are attractive.
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- For A>20 the B.E is nearly constant, it can be concluded from here that inside nucleus nucleons exert attractive forces on neighboring nucleons only in limited number i.e., nuclear forces are saturated.
- The B.E for A=56 is maximum and then gradually decrease with increase in A and ultimately for heavy nuclei of A>200 its value becomes nearly 7.6MeV that is heavy nuclei are relatively less stable. So, they split into two lighter nuclei (nuclear fission) the B/A increases and in the process tremendous energy is released.
- Similarly two lighter nuclei combine to form a relatively heavy nucleus (nuclear fusion) the B/A increases and in the process huge energy is released.

Separation Energy

- Separation energy is the amount of energy required to remove a nucleon from nucleus.
- The neutron separation energy is the amount of energy needed to remove a neutron from a nucleus.

Saturation of nuclear force

- As average B.E or B.E per nucleon i.e., B/A has approx. a constant value
- In other words we may express this as by saying B.E of nucleus is proportional to number of nucleons in the nucleus i. e B.E α A
- Now if there would have been any saturation, each of nucleon would interact with every remaining nucleon (A-1). So, total interactions of all nucleons is A (A-1). In order to count each particle single we count it by 2 i.e., the number of interacting pair= A(A-1)/2.
- And B.E would be proportional to A (A-1)/2
- i.e., B.Eα $A\frac{(A-1)}{2}$
- For heavy nuclei A<<A^2 so B.E α A^2

- This result is in sharp contrast to experimental result which is B.Eα A.
- In order to get rid of this difficulty it is necessary to assume that nucleons doesn't interact with all other nucleons but with limited number of nearby nucleons.
- If a nucleon interact with n nucleons then the total number of interactions of all nucleons is nA therefore B.Eα A.
- This situation is analogous to that of liquids or solids in which each atom is linked by chemical bonds with nearest neighbours only rather than to all other atoms. The chemical binding energy is proportional to the number of atoms present i.e., chemical B.Eα M.
- Best analogy to n.f is represented by homopolar binding like H_2 molecule.
- Saturation of nuclear forces shows that nuclear forces are short range forces i.e., force of attraction between two nucleons fall off rapidly as particles are separated.
- Sturation means a nucleon interact with limited no. of nucleons.

- Nuclear forces must be much stronger than coulomb forces. But N.F are not always attractive. As an assembly of nucleons interacting attractively by short range force would collapse to the size of order of force range i.e., if only attractive forces then nucleus would have collapsed, so there are repulsive forces. The nuclear forces turn out to be repulsive if distance between two nucleons is less than that of closest approach.

Charge independence / charge symmetry

- For light nuclei the mass no. A is approx. equal to twice the nuclear charge Z i.e., number of protons are equal or nearly equal in lighter nuclei but in heavier nuclei A is much greater than 2Z i.e., no. of neutrons increases more rapidly than number of protons.
- The equality between number of protons and number of neutrons in lighter nuclei is as if there is strong nuclear force between neutron and a proton. The stability of ${}_1H^1$ which is made up of one proton and one neutron supports the conclusion.
- If there is a strong nuclear attractive force between a neutron and a proton, it is reasonable to suppose that there are similar forces between two neutrons and between two protons i.e., force between two neutrons must be nearly equal to that between two neutrons.

- If attraction b/w two neutrons were greater than b/w two protons then lighter nuclei would have more neutron than that of protons. Similar argument shows that force b/w two protons cant be greater than neutron.
- In a heavy nuclei there is increase in coulomb repulsion between protons, the nucleus needed to overcome this effect of electrostatic effect. This is over comed by increase in no of neutrons compared to the no of protons i.e., $N>Z$. The excess number of neutrons confirms the attraction between neutrons. As coulomb energy increases the ratio of no of neutrons to that of no. of protons increases from 1.0 to 1.6 gradually.

- Thus, it is clear that there is force of attraction b/w any two types of nucleons. The forces are of three kinds n-n, n-p, and p-p. The n-n and p-p is equal or very nearly equal in magnitude but not greater than n-p forces.
- The two possibilities are expressed as

a) n-p~ n-n ~p-p (charge independence).

b)n-n<n-p; p-p<n-p; p~p (charge symmetry).

The first assumption is called as hypothesis of charge independence i.e., magnitude of force between two nucleons is independent of charge on nucleons.

The second assumption is called as hypothesis of charge symmetry.

- Consider two mirror $_1H^3$nuclei $_2He^3$ and. The former contains 1p and 2 n while later contains 2p ad 1n. In $_1H^3$ we have two n-p and one n-n bonds and in $_2He^3$ we have two n-p and one p-p bond

B.E ($_1H^3$)= 8.482 MeV and B.E ($_2He^3$)=7.711 MeV. So, difference in binding energy =0.771MeV which is small compared to either binding energy. The difference may be due to coulomb repulsion b/w p-p in $_1H^3$. Since there is no coulomb force in $_1H^3$ it possess more B.E.

Thus n-n force in $_1H^3$ and p-p force in $_2He^3$ aren't vey different.

If $_2He^3$ is assumed to be spherical with radius R_e its coulomb energy is

Since E_c=3/5 Z (Z-1) e^2/R

Coulomb energy of = 6/5 e^2/R_e =0.771 MeV

This equation is solved with R_e= 2.34x10^{-13} cm = 2.3 fm.

This hypothesis that n-n forces in $_1H^3$ is equal to p-p in $_2He^3$ is therefore consistent

- Although mirror nuclei provide evidence in support of charge symmetry of nuclear forces but they didn't tell anything about magnitude of n-p force compared to p-p and n-n force.
- For this consider, three isobars ${}_{4}^{10}Be_6$, ${}_{5}^{10}B_5$, ${}_{6}^{10}C_4$
- Because of equality of n-n and p-p forces Be and C should behave in a similar way as mirror nuclei do. Their ground state energy should be obtained from difference in coulomb energy and the energy equivalent to difference in mass , b/w proton and neutron. This argument can be extended to B but only if n-p forces is equal to n-n and p-p forces. Since n-p bonds in B should differ from no. of n-p bonds in Be and C. It turns to be out that certain excited states of B should bear a close relation to ground state of Be and C, there is good evidence that excited states exist.
- A similar argument can be applied to ${}_{6}^{10}C_8$, ${}_{7}^{14}N_7$, ${}_{8}^{14}O_6$

Deutron

- Of three possible states of two nucleons system dineutron, diproton and deutron only deutron is known to be stable. The deutron binding energy is B.E ($_1{}^2d_1$)= 2.225 MeV.
- So, B.E per nucleon B/A is 2.225/2= 1.1125 MeV/Nucleon
- Since B.E of deutron is small thus deutron is very loosely bound state.
- The energy of gamma ray that will break the bond between the neutron and proton is only $E\gamma>2.225$ MeV. This is called photodisintegration
- $\gamma+d=p+n$ provided $E\gamma>2.225$MeV.
- The total angular momentum is defined as $L=\Sigma l_i$ and $S = \Sigma S$
- In case of deutron the total angular momentum has been determined as J=1.

- There are only two interacting particles proton and neutron both are fermions. Therefore spin angular momentum S=0,1.
- The lowest state is one with smallest allowed values of orbital angular momentum then;
- If l=0, then spin of g.s of deutron is S=1.
- So, g.s of deutron is 3S1.
- This means that spins of neutrons and protons are parallel and deutron is in triplet state.
- S=0 i.e., singlet spin state isnot possible because in this state the spins of protons and neutrons are anti-parllel.
- Then n-p interactions are weaker in this state than S=1 state.
- Thus, only S=1, triplet state leads to bound state.
- It is clear from this the N.F are spin dependent.
- $V_{n\text{-}p}(S=0) \neq V_{n\text{-}p}(S=1)$.